Multipoles in Galaxy Surveys

Journeying to Higher Orders

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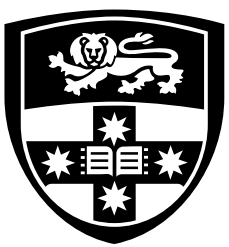
Novbember 8, 2024



THE UNIVERSITY OF SYDNEY

HDR Symposium

November 2024

Agenda

Introduction

Method

Results

Discussion & Conclusions

Introduction

A refresher...

The Kinematic Dipole

- Imprinted on the CMB is a dipole ($\Delta T/T \approx 10^{-3}$).

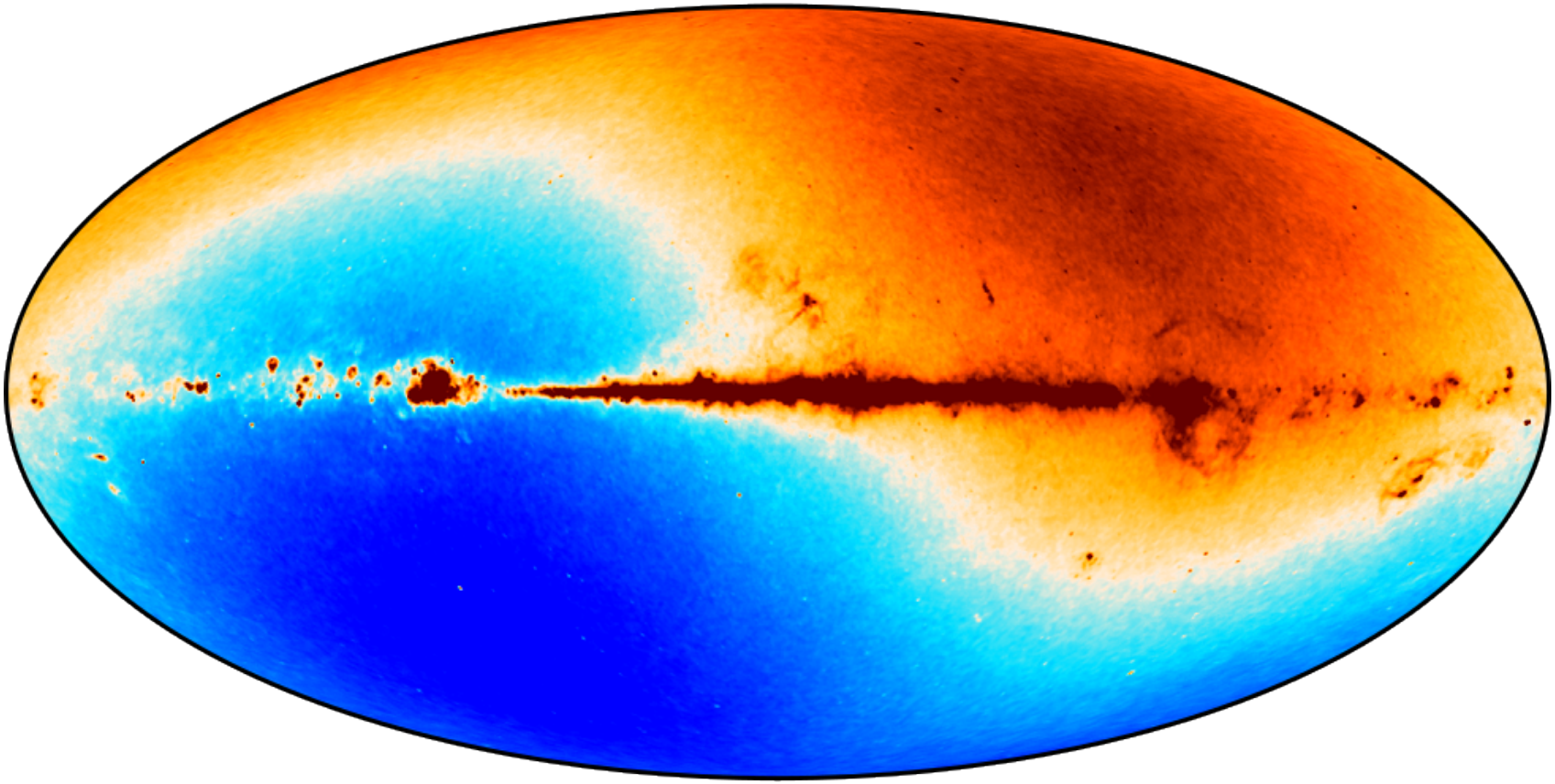


Figure 1: CMB temperature map (dipole included; BeyondPlanck). ★: dipole direction.

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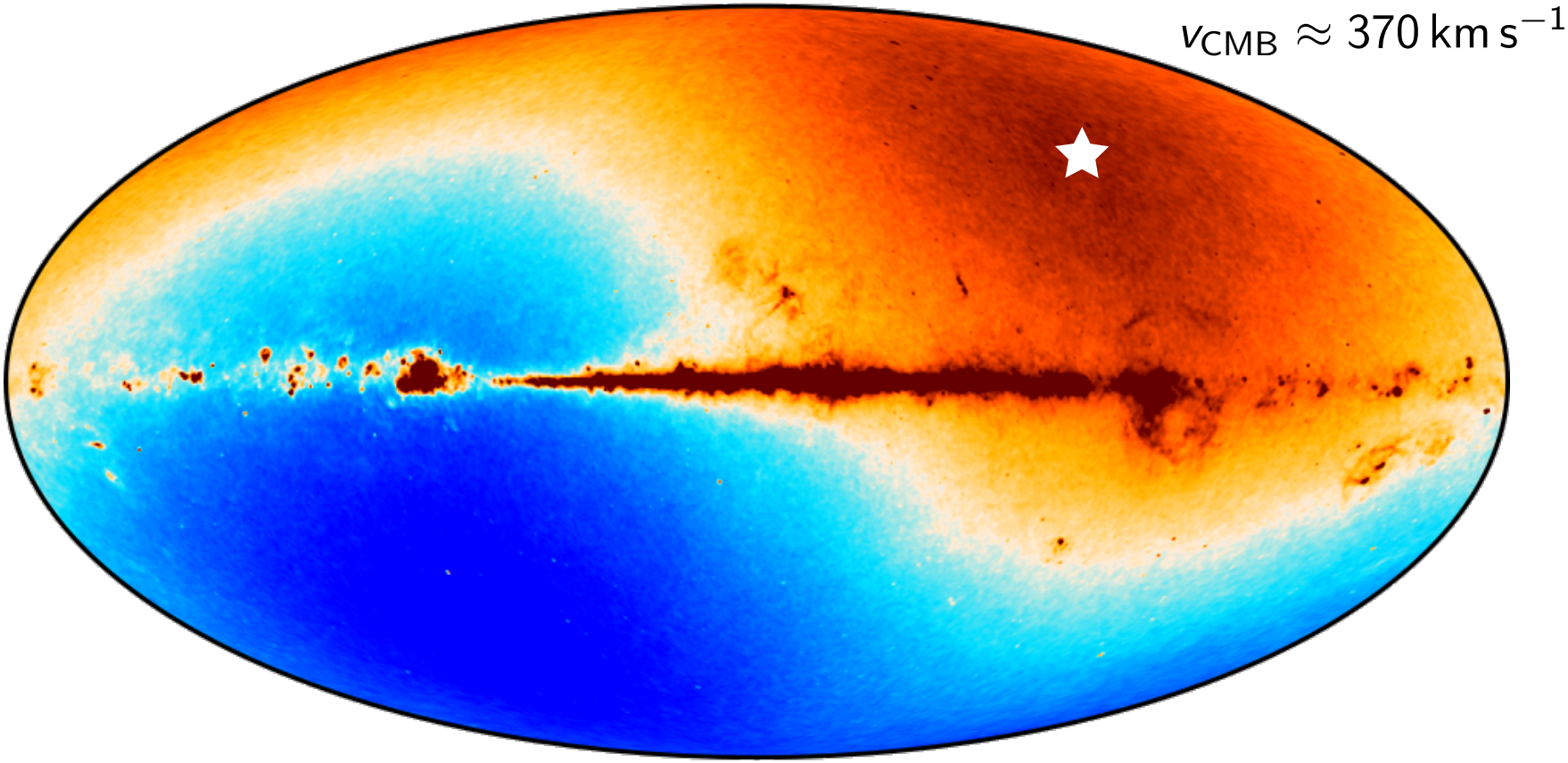

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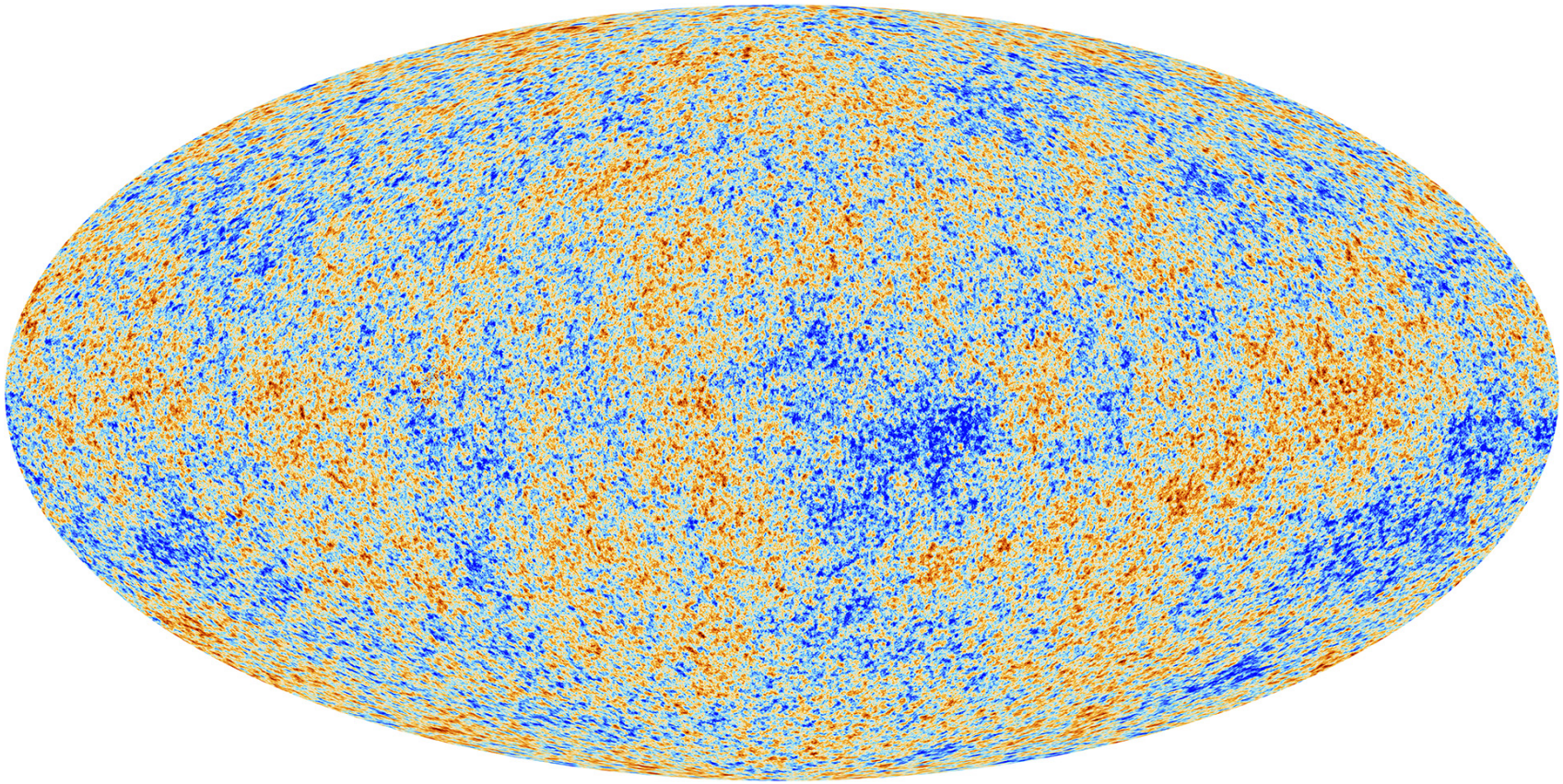
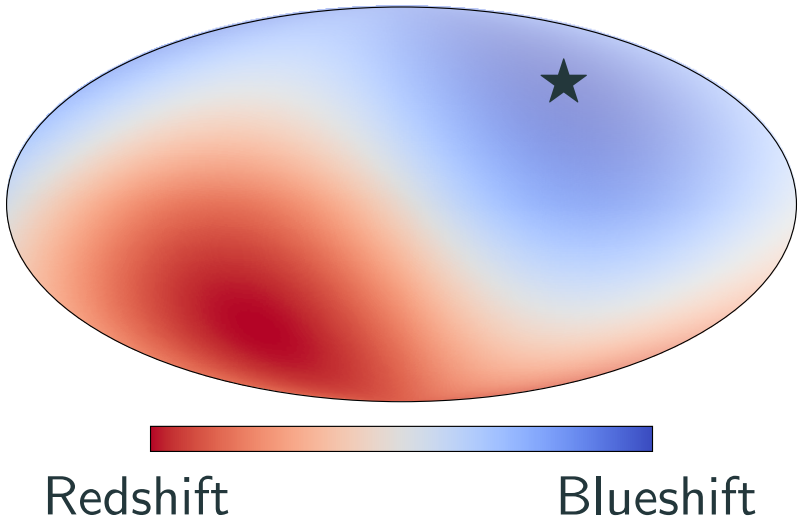


Figure 1: CMB temperature map (dipole excluded; Planck).

The Ellis & Baldwin (1984) Dipole



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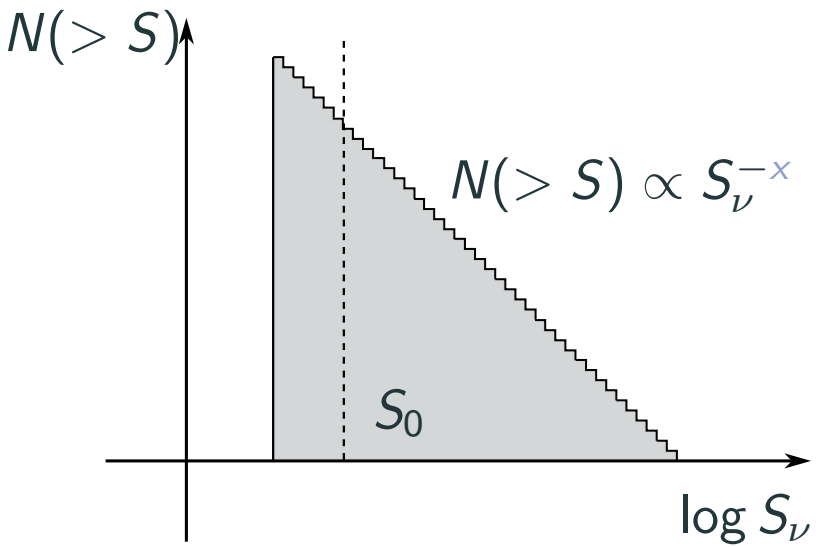

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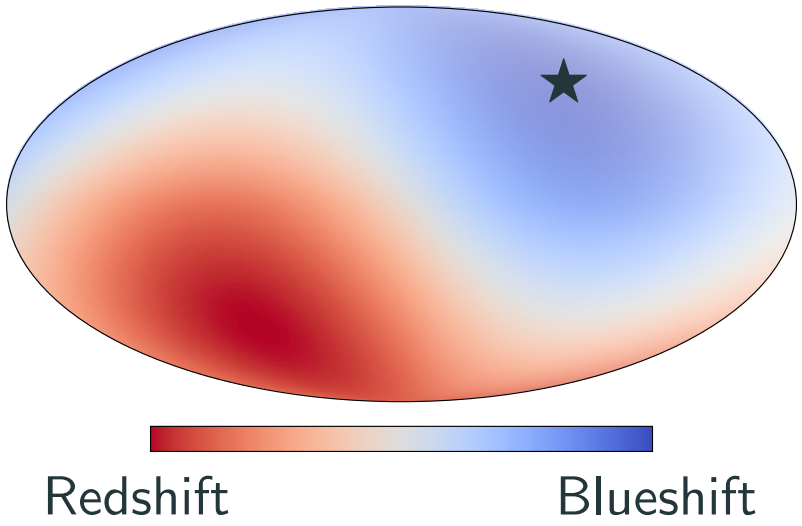

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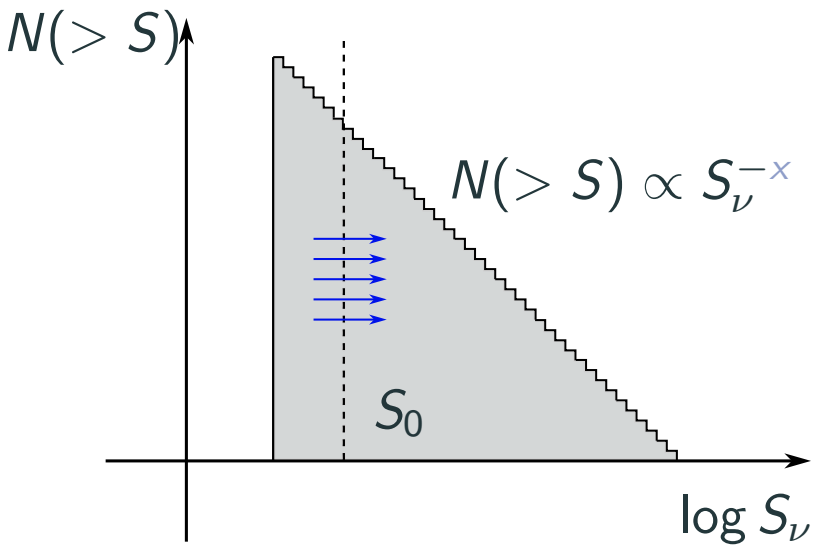

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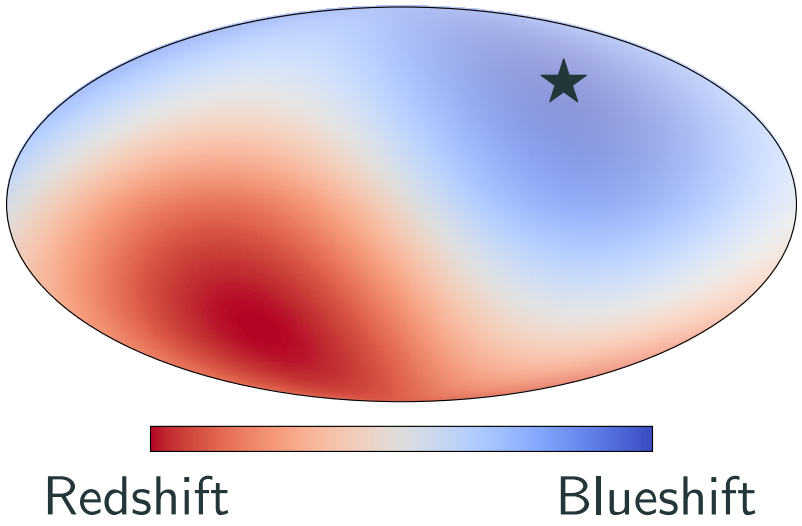

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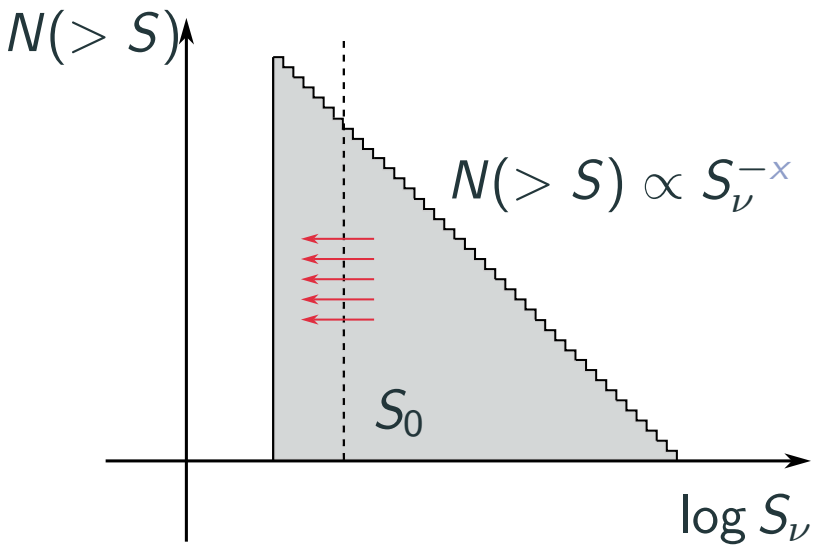

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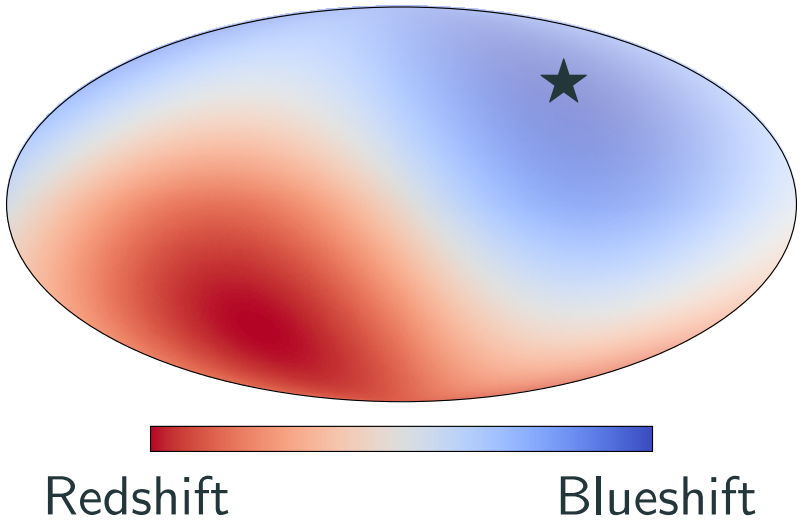

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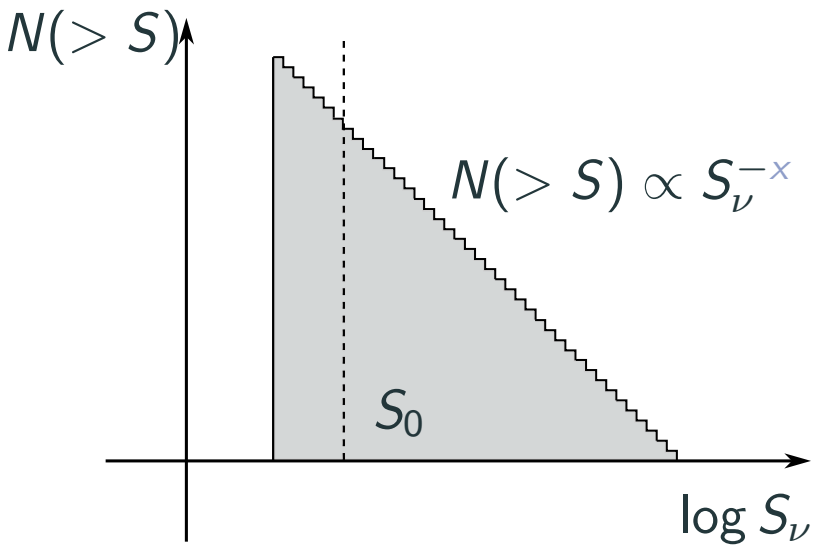

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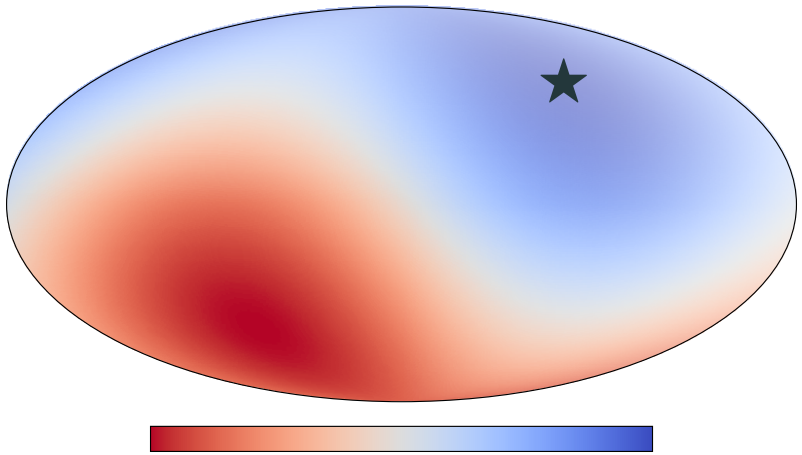

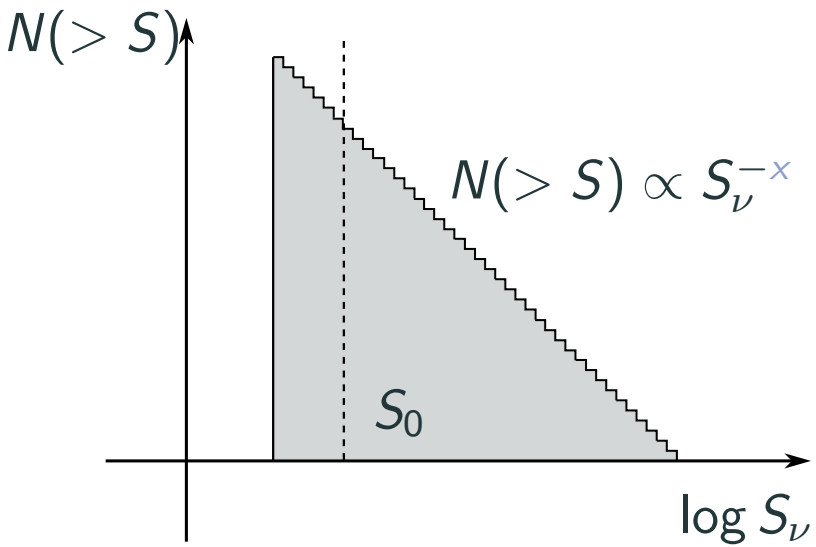

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$$\mathcal{D}_{\mathrm{CMB}} = [2 + x\,(1 + \alpha)]\,\frac{v_{\mathrm{CMB}}}{c}.$$

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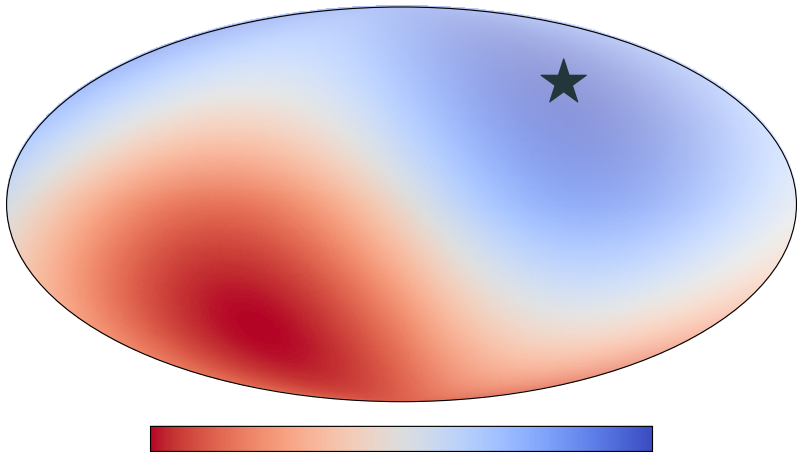

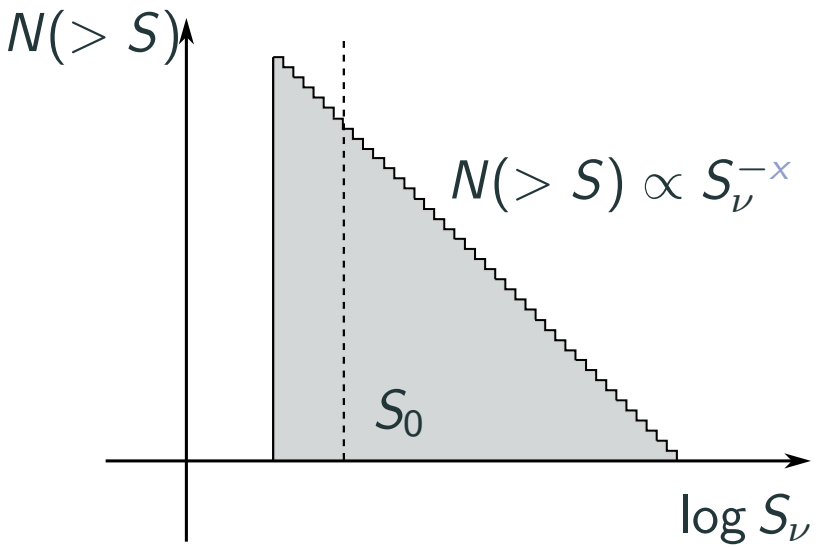

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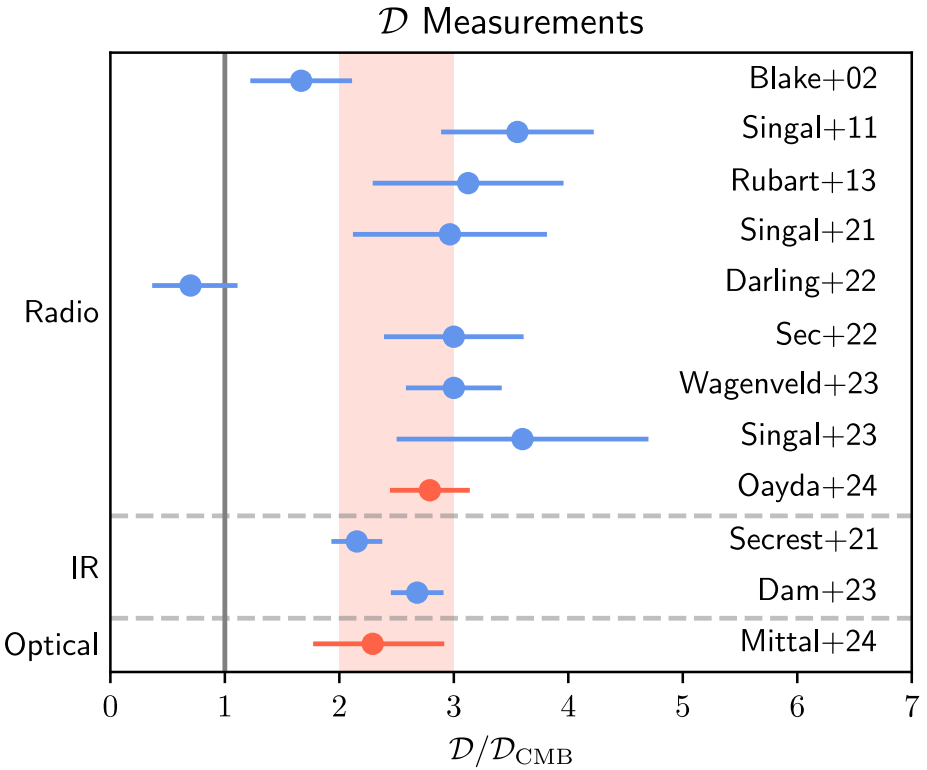
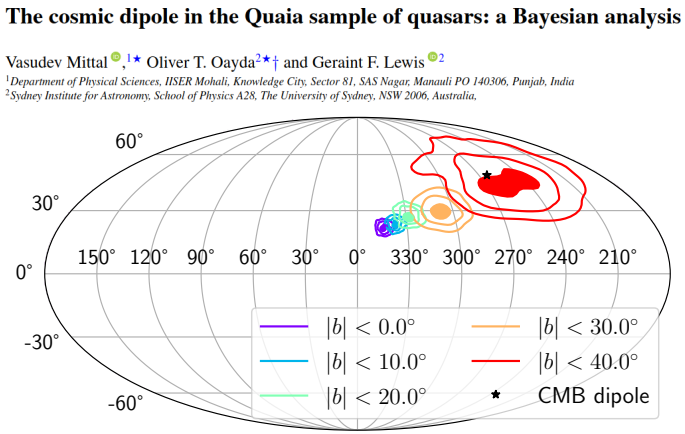
Typical values are 0.004 – 0.007. This is a *0.5% effect!*

The Dipole Tension





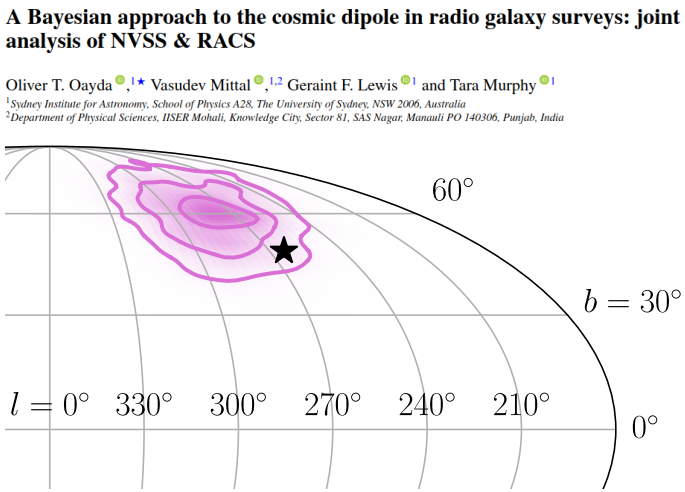
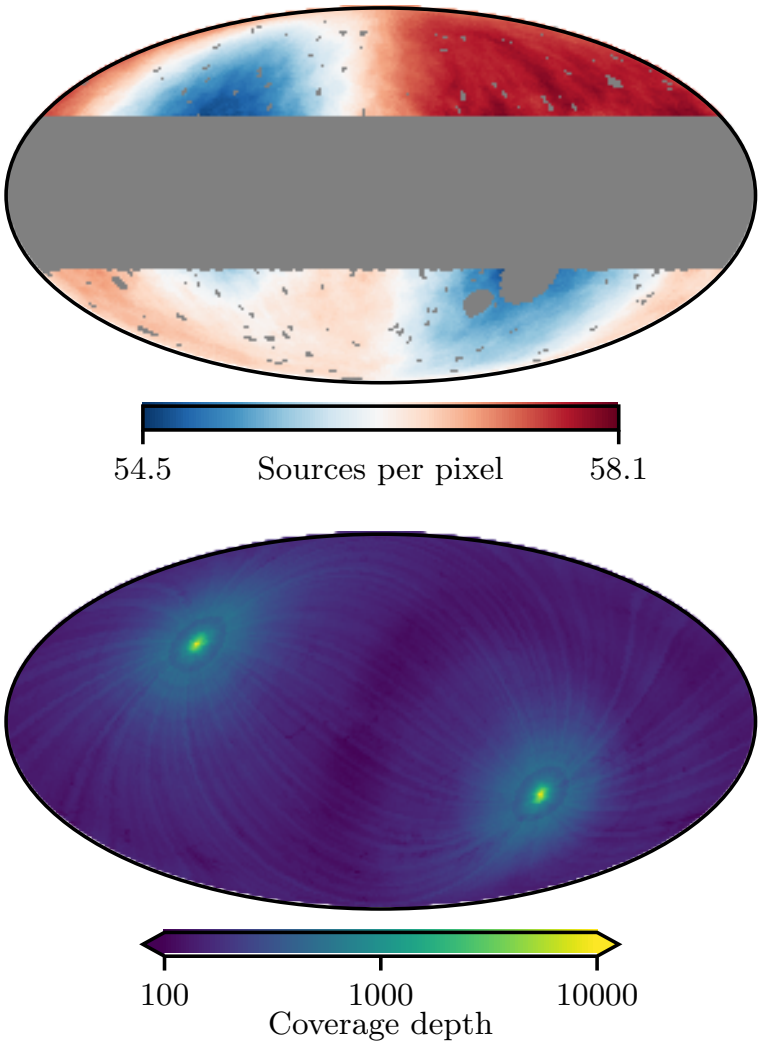


Figure 2: *Top:* Literature values for $\mathcal{D}$ (1σ); our studies in red. *Right:* Dipole direction results from our studies.

What is going on here? Is this a genuine tension or are there systematics?

CatWISE2020 and Multipoles

CatWISE2020 has a known ecliptic bias
$\Longrightarrow$ quadrupole ($\ell = 2$).

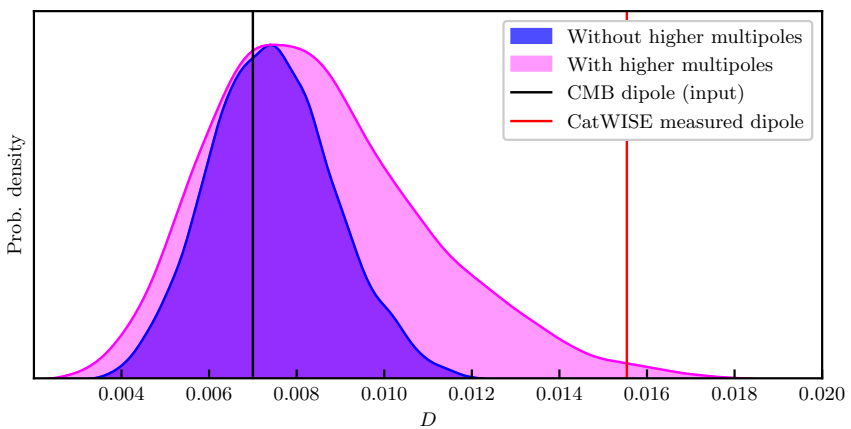


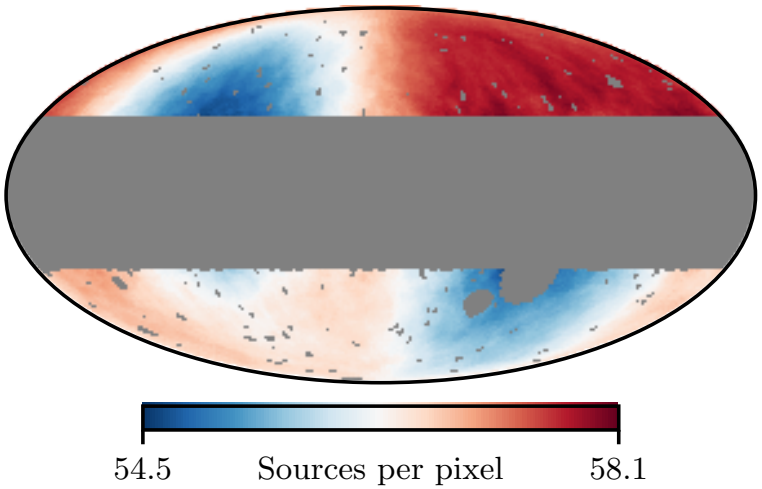
CatWISE2020 and Multipoles

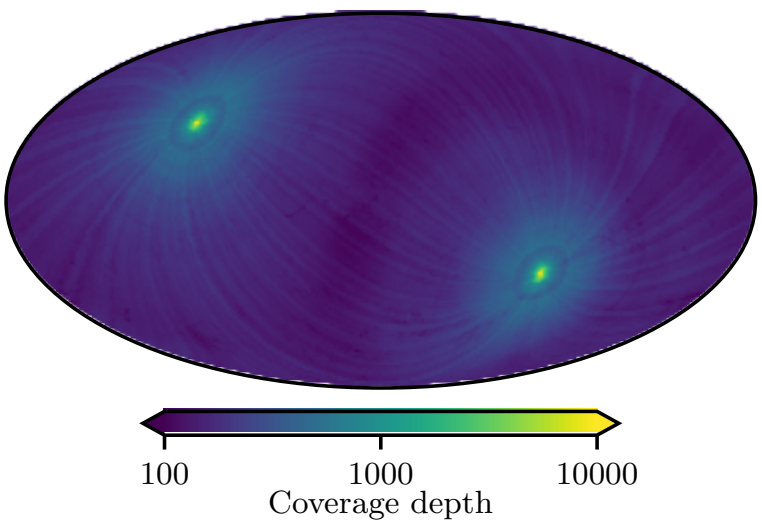
CatWISE2020 has a known ecliptic bias $\implies$ quadrupole ($\ell = 2$).

But what if there are additional, higher order multipoles on a masked sky? Mode coupling; power leakage.

Figure 3: Plots from Abghari et al. (2024). *Right:* Smoothed CatWISE density map & scanning law. *Bottom:* Effect of additional multipoles on $\mathcal{D}$.







Let's investigate...

Method

Defining a Dipole

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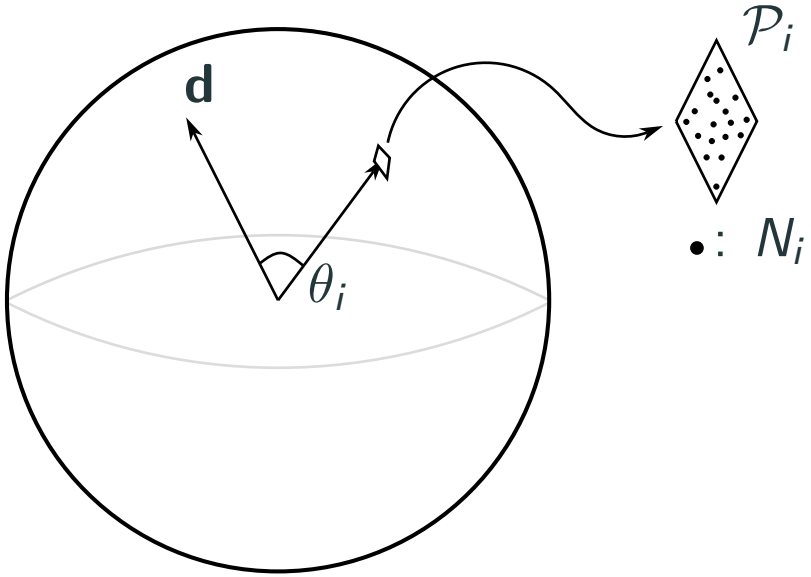
What does it take to define a dipole?



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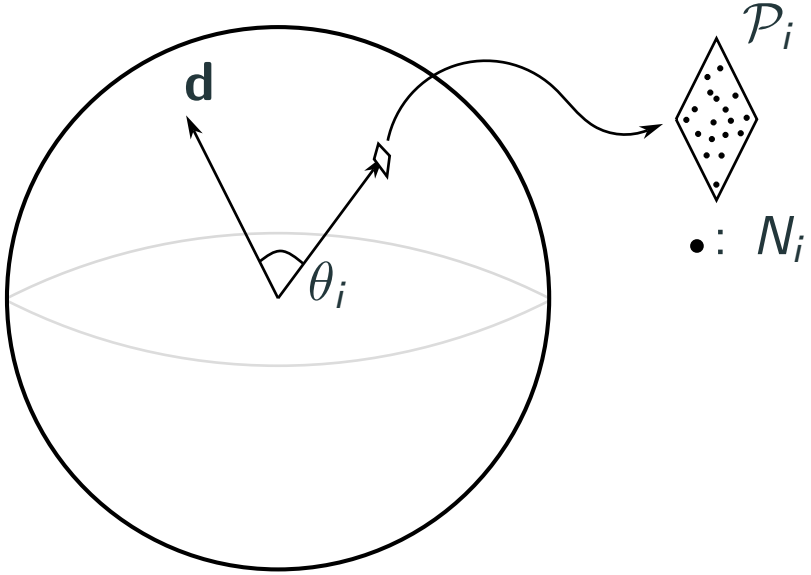
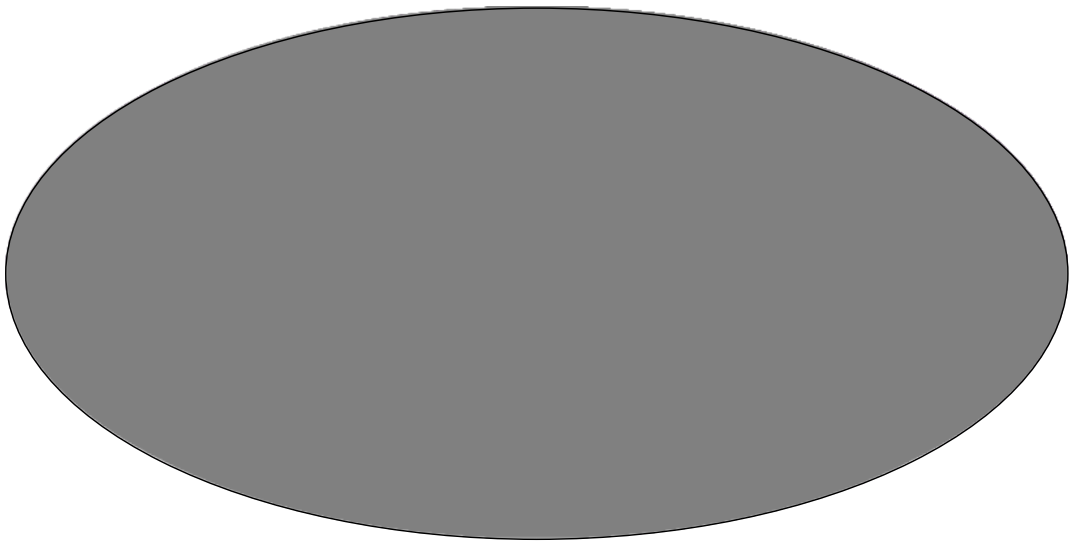
We need **three** parameters: an amplitude and two for the direction.

$$\begin{aligned}\mathbb{E}[N_i] &= \bar{N}(1 + \mathbf{d} \cdot \hat{\mathbf{p}}) \\ &= \bar{N}(1 + \mathcal{D} \cos\theta_i)\end{aligned}$$



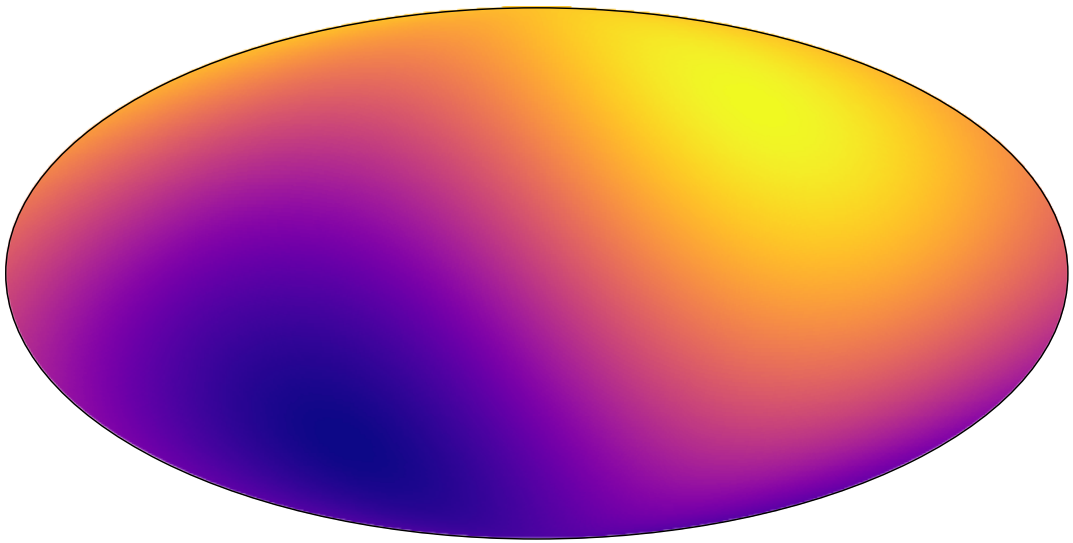
Counting Above $\ell = 1$

$\ell = 0$: monopole,
1 parameter



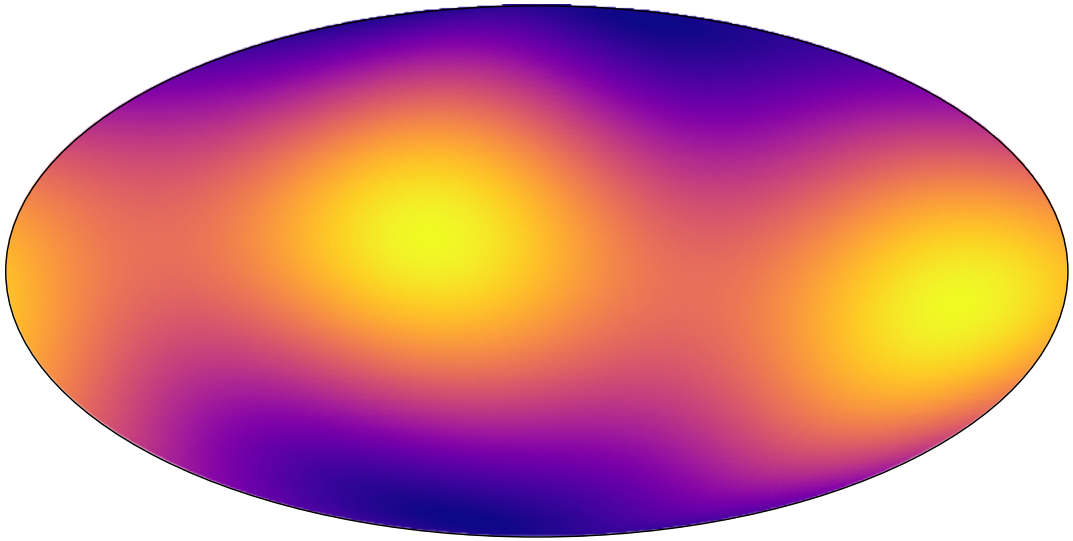
Counting Above $\ell = 1$

$\ell = 1$: dipole,
3 parameters



Counting Above $\ell = 1$

$\ell = 2$: quadrupole,
5 parameters



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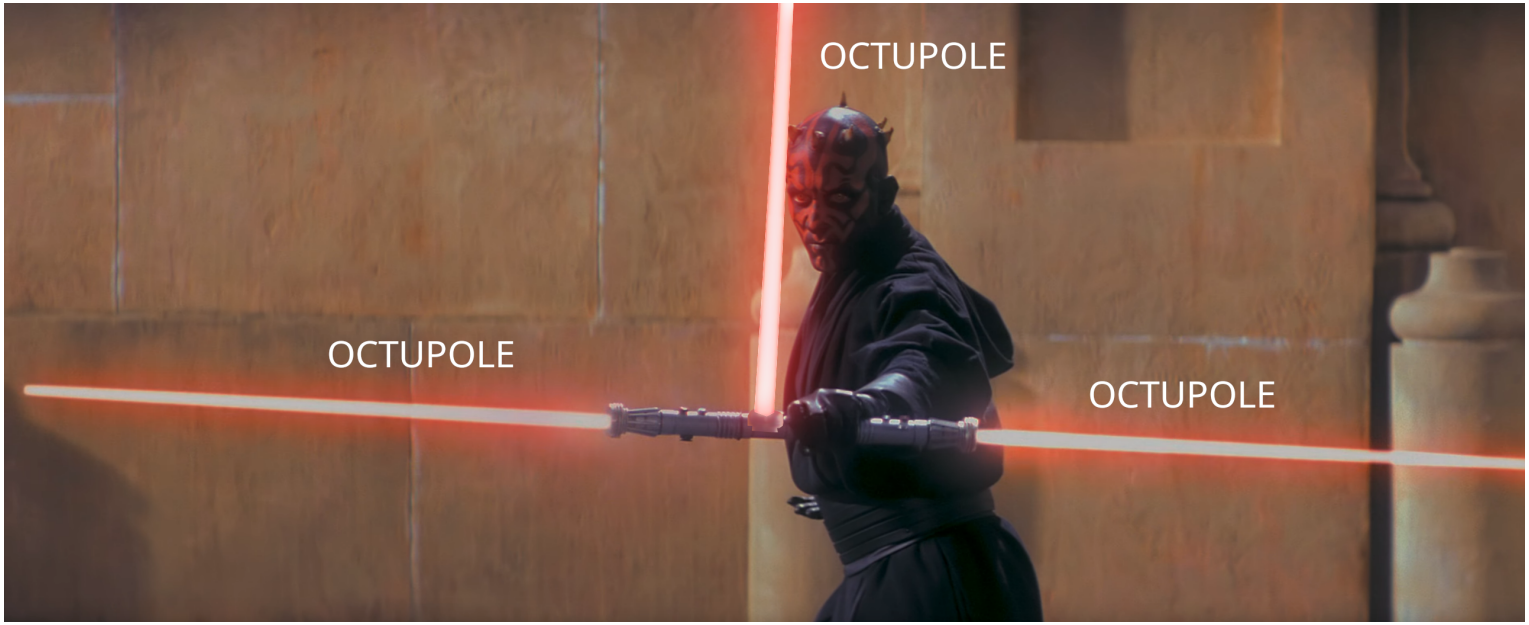
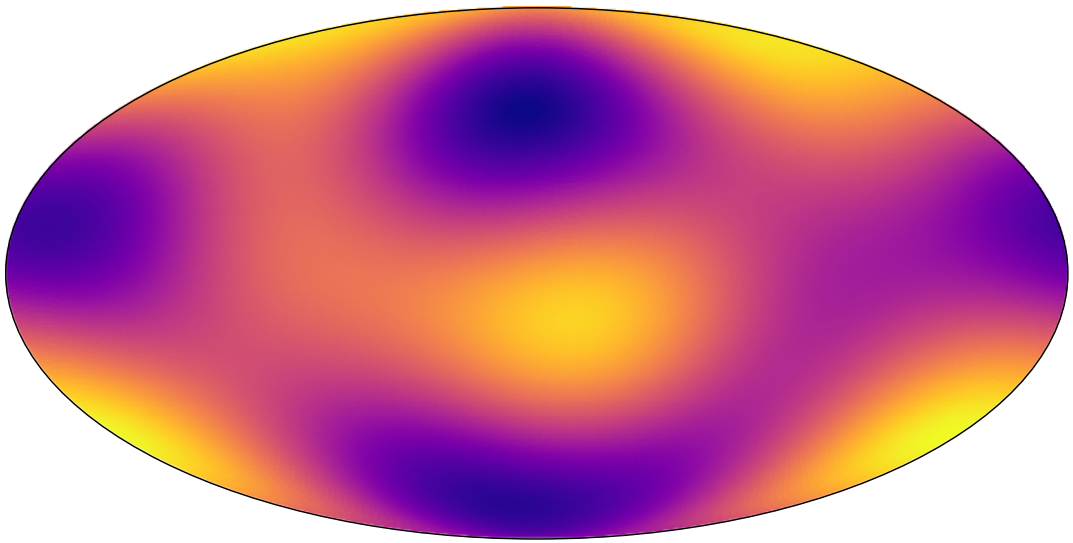


$\ell = 3$: octupole,
7 parameters



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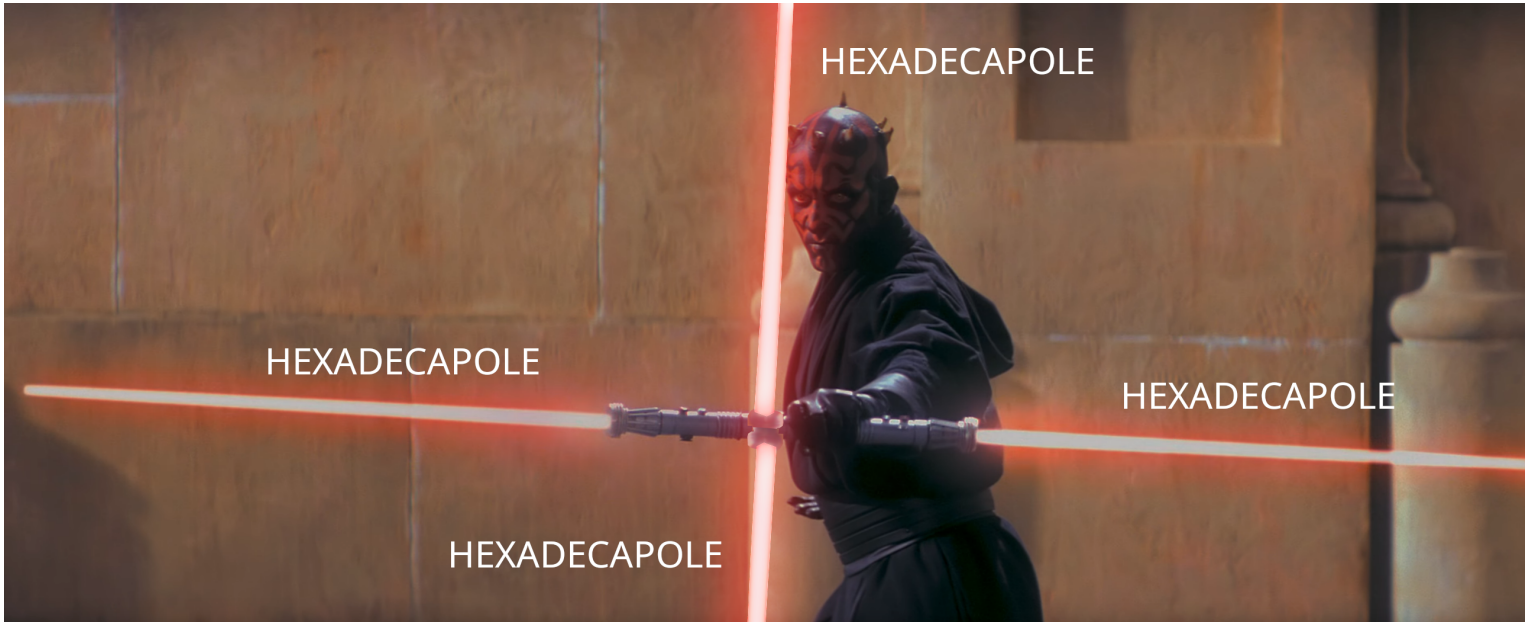
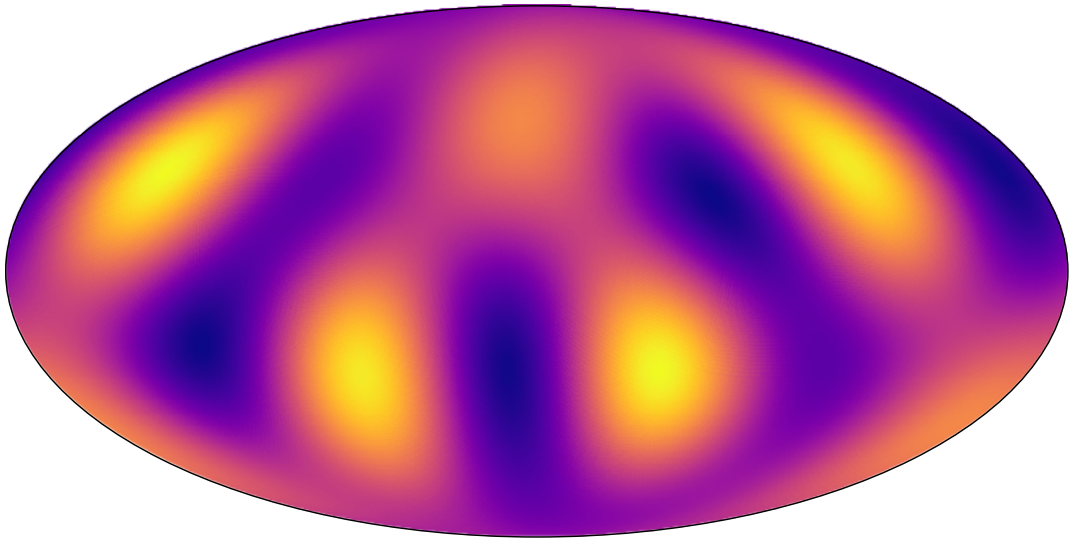
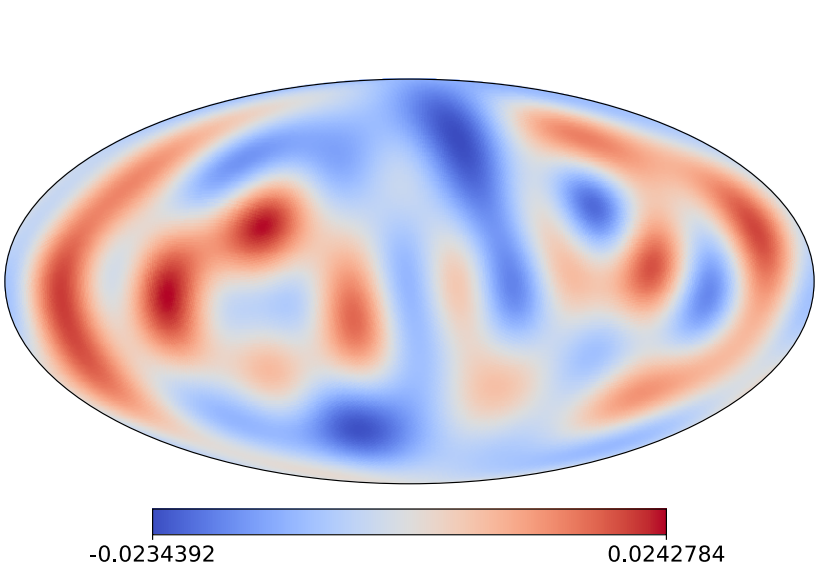


$\ell = 4$: hexadecapole,
9 parameters



We can combine multipoles!



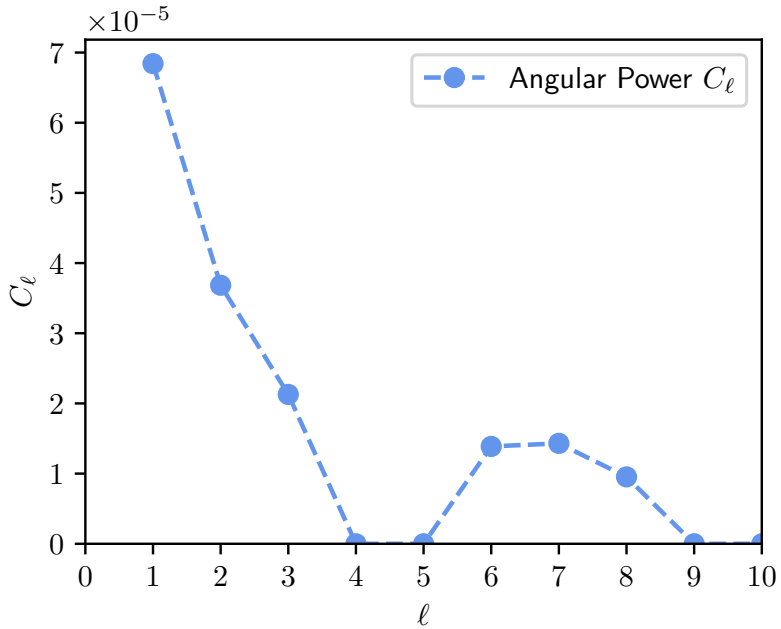


Figure 4: *Left:* Multipole signal $f_{\text{mult.}}$ made by summing contributions from the $\ell = 1$, $\ell = 2$, $\ell = 3$, $\ell = 6$, $\ell = 7$ and $\ell = 8$ multipoles. *Right:* Power spectrum computed from the map above (Oayda et al., submitted).

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This technique ('traceless symmetric tensor method') has some pedigree.

Using Simulations

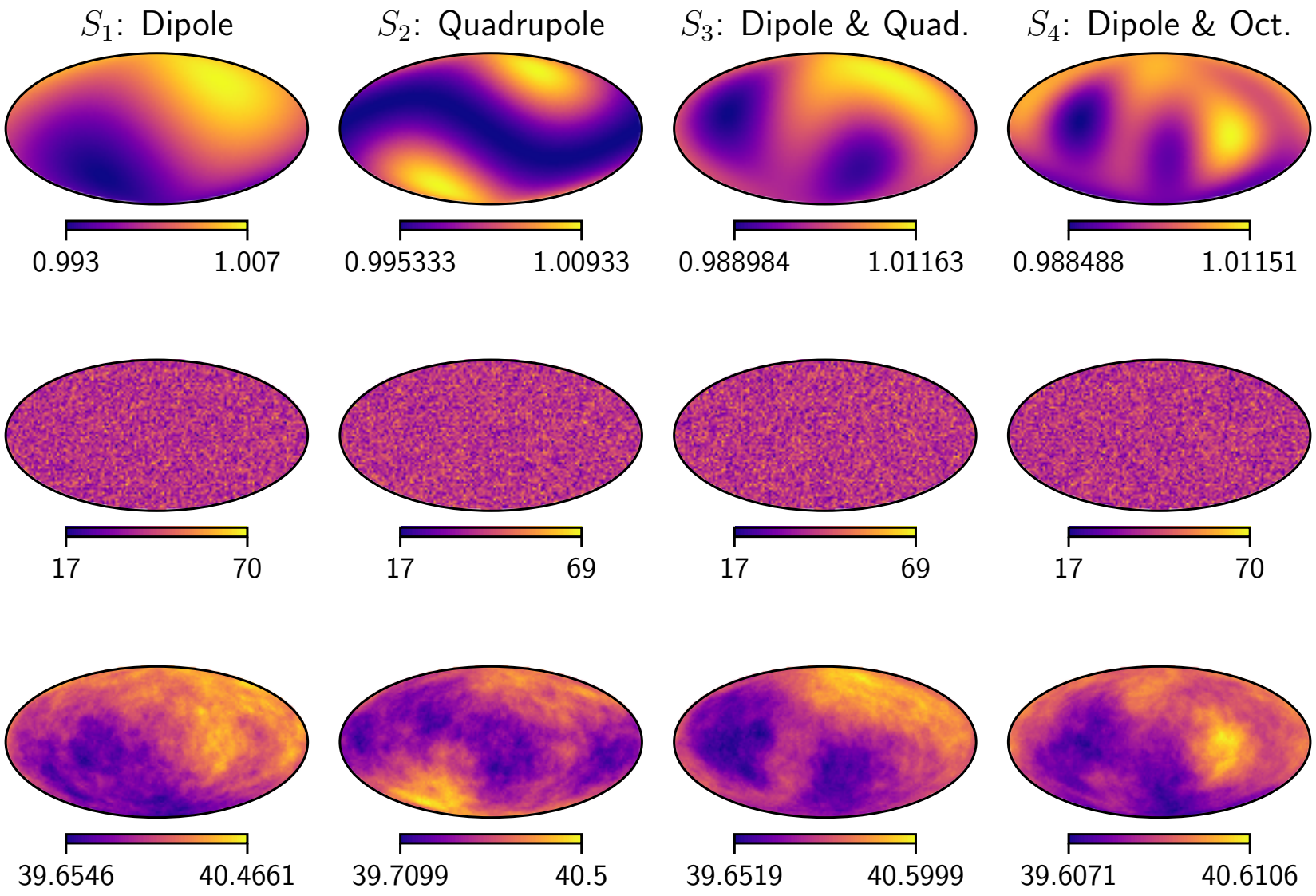


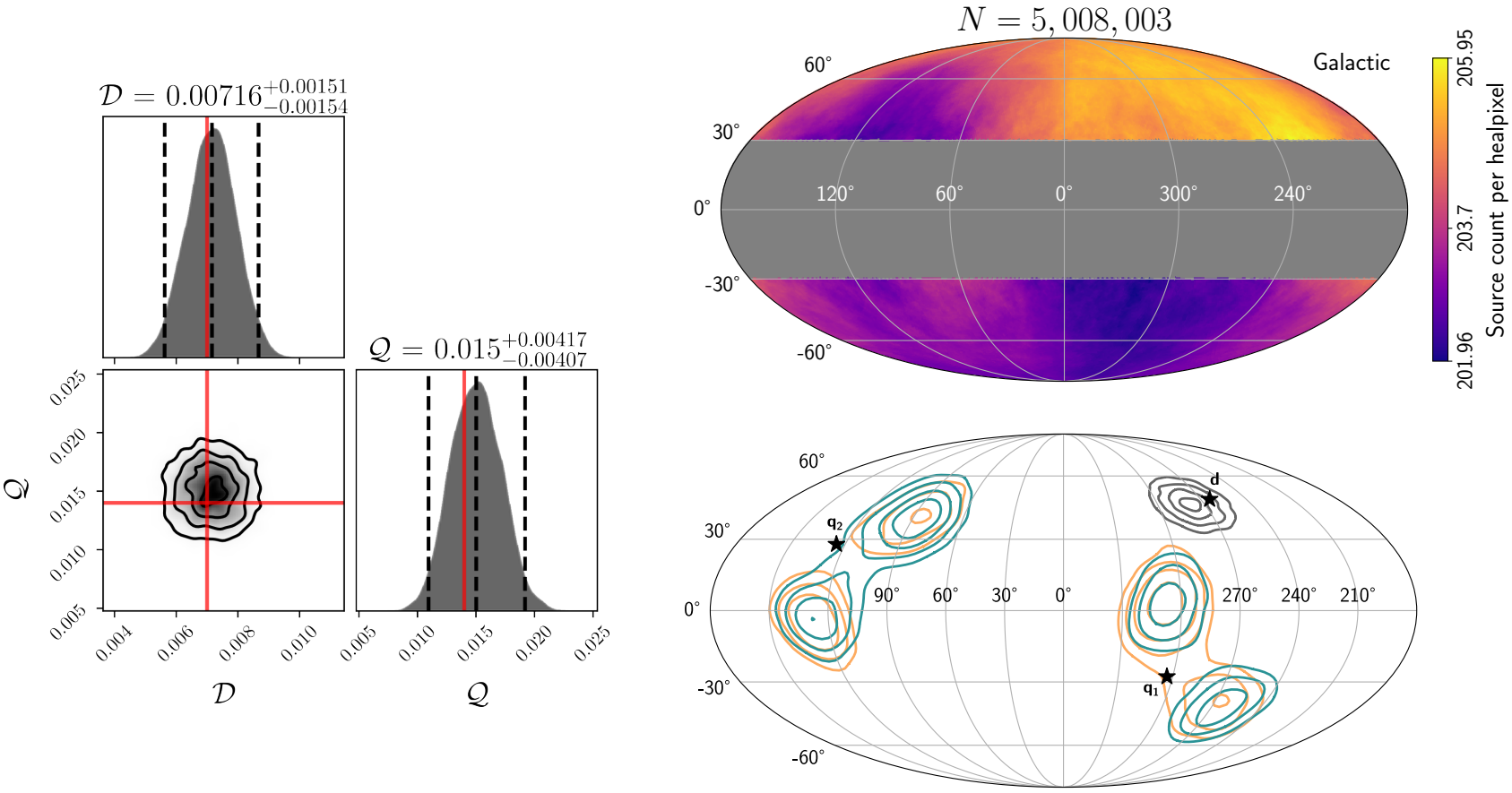
Figure 5: Sample templates constructed and tested (Oayda et al., submitted).

Question: can we disentangle dipoles & higher order multipoles on masked skies?

Results

Dipole & Quadrupole

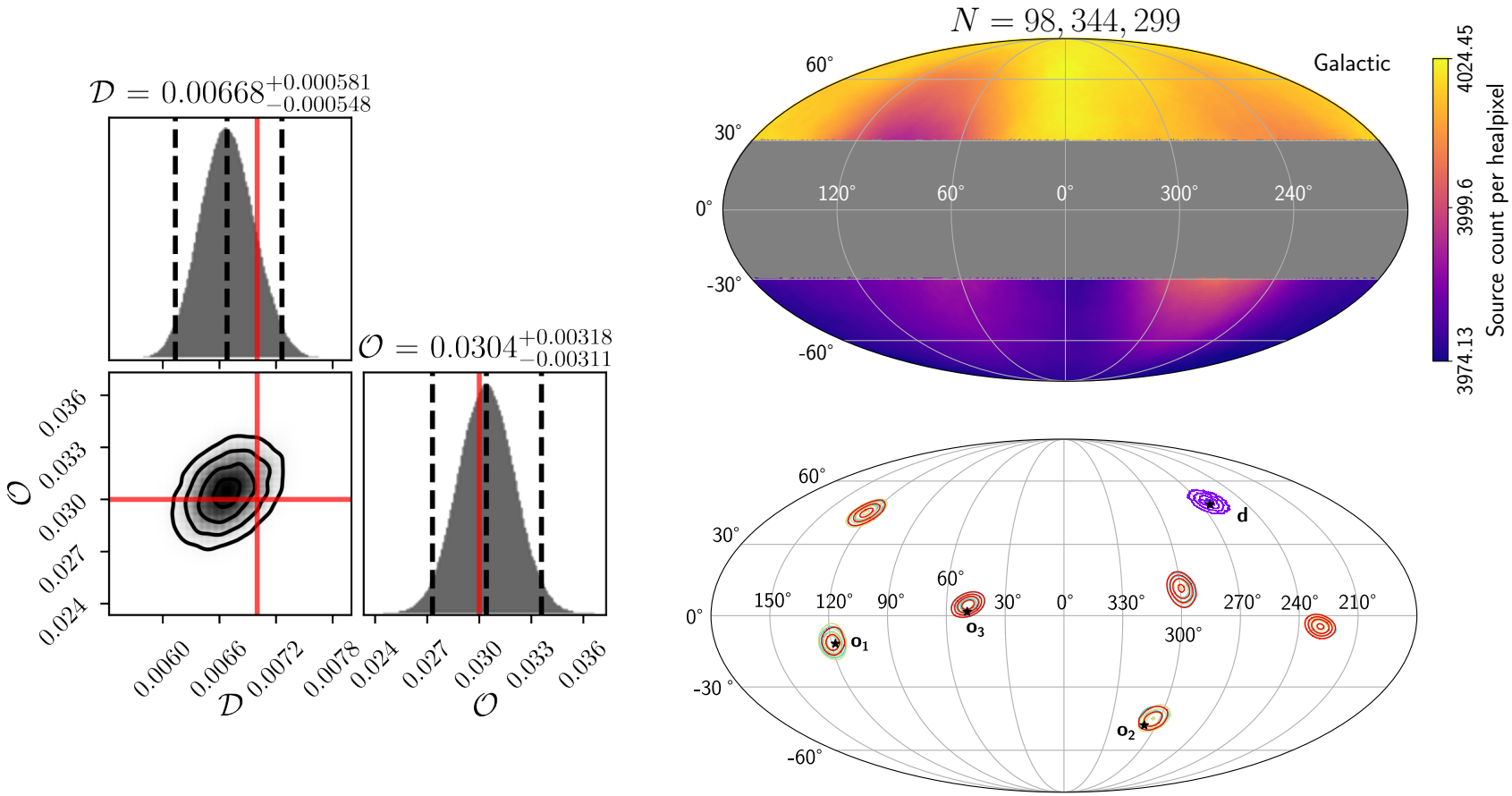
Figure 6: Results from the fit to sample S_3 (Oayda et al., submitted).



The dipole & quadrupole are disentangled from each other!

Dipole & Octupole

Figure 7: Results from the fit to sample S_4 (Oayda et al., submitted).



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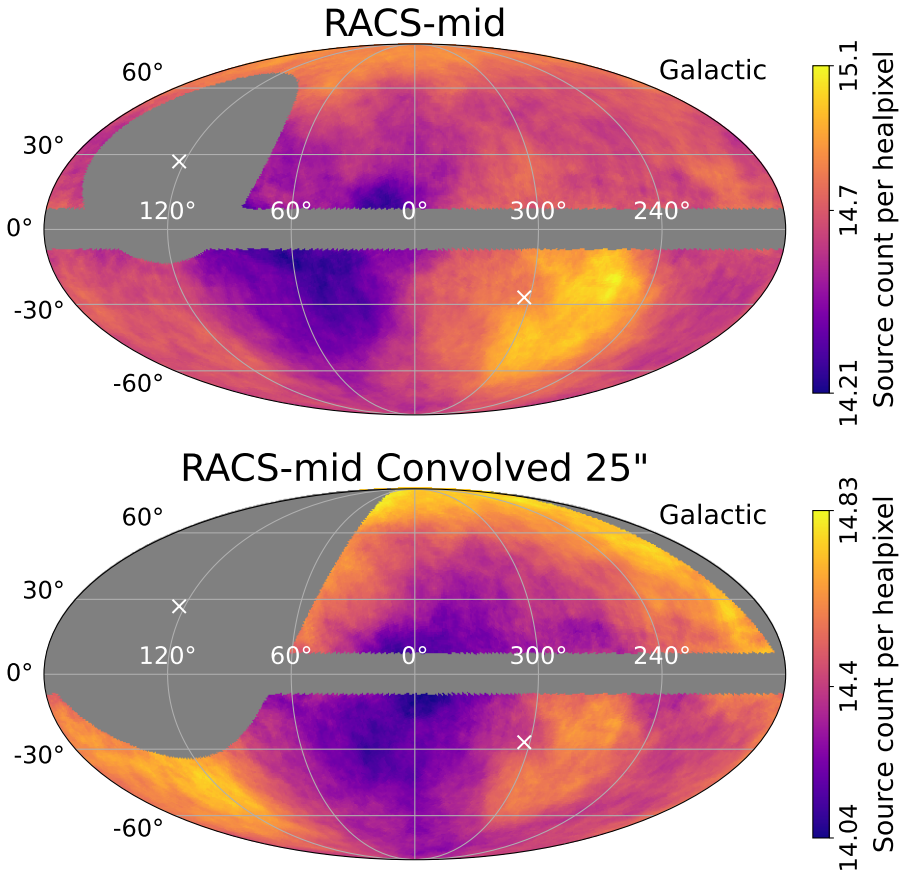

Figure 8: $10\,\mathrm{mJy} < S_{\mathrm{int.}} < 1000\,\mathrm{mJy}$. Smoothed RACS-mid maps (Galactic coordinates). *Top:* RACS-mid. *Bottom:* RACS-mid convolved to 25" common resolution.

Conclusion

This will help us asses the robustness of the dipole tension.

In so doing, we interrogate the basis of the cosmological principle.

Any questions?

References

Abghari A., Bunn E. F., Hergt L. T., Li B., Scott D., Sullivan R. M., Wei D., 2024, arXiv e-prints, p. arXiv:2405.09762

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